



Cambridge International AS & A Level

CANDIDATE
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MATHEMATICS

9709/31

Paper 3 Pure Mathematics 3

May/June 2020

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Blank pages are indicated.

1 Find the set of values of x for which $2(3^{1-2x}) < 5^x$. Give your answer in a simplified exact form. [4]

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2 (a) Expand $(2 - 3x)^{-2}$ in ascending powers of x , up to and including the term in x^2 , simplifying the coefficients. [4]

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(b) State the set of values of x for which the expansion is valid. [1]

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4 The curve with equation $y = e^{2x}(\sin x + 3 \cos x)$ has a stationary point in the interval $0 \leq x \leq \pi$.

(a) Find the x -coordinate of this point, giving your answer correct to 2 decimal places. [4]

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(b) Determine whether the stationary point is a maximum or a minimum. [2]

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5 (a) Find the quotient and remainder when $2x^3 - x^2 + 6x + 3$ is divided by $x^2 + 3$. [3]

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(b) Using your answer to part (a), find the exact value of $\int_1^3 \frac{2x^3 - x^2 + 6x + 3}{x^2 + 3} dx$. [5]

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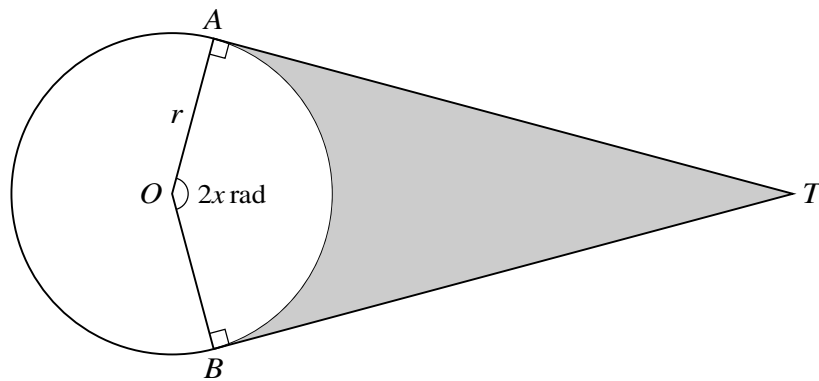
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The diagram shows a circle with centre O and radius r . The tangents to the circle at the points A and B meet at T , and angle AOB is $2x$ radians. The shaded region is bounded by the tangents AT and BT , and by the minor arc AB . The area of the shaded region is equal to the area of the circle.

- (a) Show that x satisfies the equation $\tan x = \pi + x$. [3]

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- (b) This equation has one root in the interval $0 < x < \frac{1}{2}\pi$. Verify by calculation that this root lies between 1 and 1.4. [2]

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- (c) Use the iterative formula

$$x_{n+1} = \tan^{-1}(\pi + x_n)$$

to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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7 Let $f(x) = \frac{\cos x}{1 + \sin x}$.

(a) Show that $f'(x) < 0$ for all x in the interval $-\frac{1}{2}\pi < x < \frac{3}{2}\pi$. [4]

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- (b) Find $\int_{\frac{1}{6}\pi}^{\frac{1}{2}\pi} f(x) dx$. Give your answer in a simplified exact form. [4]

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8 A certain curve is such that its gradient at a point (x, y) is proportional to $\frac{y}{x\sqrt{x}}$. The curve passes through the points with coordinates $(1, 1)$ and $(4, e)$.

(a) By setting up and solving a differential equation, find the equation of the curve, expressing y in terms of x . [8]

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(b) Describe what happens to y as x tends to infinity. [1]

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- 9 With respect to the origin O , the vertices of a triangle ABC have position vectors

$$\vec{OA} = 2\mathbf{i} + 5\mathbf{k}, \quad \vec{OB} = 3\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} \quad \text{and} \quad \vec{OC} = \mathbf{i} + \mathbf{j} + \mathbf{k}.$$

- (a) Using a scalar product, show that angle ABC is a right angle. [3]

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- (b) Show that triangle ABC is isosceles. [2]

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(c) Find the exact length of the perpendicular from O to the line through B and C .

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10 (a) The complex number u is defined by $u = \frac{3i}{a+2i}$, where a is real.

(i) Express u in the Cartesian form $x + iy$, where x and y are in terms of a . [3]

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(ii) Find the exact value of a for which $\arg u^* = \frac{1}{3}\pi$. [3]

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- (b) (i) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - 2i| \leq |z - 1 - i|$ and $|z - 2 - i| \leq 2$. [4]

- (ii) Calculate the least value of $\arg z$ for points in this region. [2]

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Additional Page

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Lined area for writing answers, consisting of multiple horizontal dotted lines.

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